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An Optimization Method for the Determination of the Important Flutter Modes

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An optimization method for the determination of the dominant flutter modes is presented in this paper. The method is based on the minimization of the quadratic values of subdeterminants derived from the equations of motion. The effectiveness of the method is illustrated by seven numerical examples.

Introduction

THE flutter analysis of modern aircraft requires the consideration of a large number of degrees of freedom. Once the flutter is determined and found to lie within the flight envelope, attempts are normally made to reduce the number of degrees of freedom by tracing the modes (or degrees of freedom) which are responsible for the flutter instability. Experience shows that in most cases, flutter is essentially caused by two degrees of freedom and more rarely by three degrees of freedom (or more). Therefore, the reduction in the number of degrees of freedom is considerable and makes it easier to study the effects of variation of coefficients in the equations of motion on the flutter speed.

Several methods have been proposed for the determination of the important flutter modes,[1-4] but all of them appear to have limitations which prevent their adoption as standard methods for the aeroelasticians. A short description of these methods is presented in the following:

1) The frequency coalescence method. This method is based on the fact that, in many instances, flutter is caused by either the frequency coalescence or by the near coalescence of two neighboring modes. The plots of the frequencies vs flight speed are searched for a frequency coalescence around the flutter speed and flutter frequency of the system. The coalescing modes are assumed to represent the important flutter modes and an analysis of this reduced system is then performed to verify that its flutter speed is very near the speed yielded by the expanded system. This method is useful, but is often incapable of tracing the important flutter modes. Such examples are presented in this paper and are discussed in later sections of this paper.

2) The systematic order reduction method. This method[1] is based on a systematic reduction of the order of the system by a single degree of freedom each time, and on the solution for flutter of the reduced system. This procedure is repeated until the smallest number of degrees of freedom is obtained which yields the same flutter speed as the original system. The main drawback of this method lies in the considerable computational labor involved in the determination of the important flutter modes.

3) The energy methods. The energy methods are based on expressions related to the energy input into the system during one cycle of harmonic oscillation. Reference 2 suggests a method for the detection of the important flutter modes, based on the energy compatibility equation developed in Ref. 5. All five flutter examples treated in Ref. 2 relate to undamped systems, that is, to systems with no damping at all of either a structural or an aerodynamic nature. However, when attempting to treat systems with damping, the energy compatibility equation fails to yield the important flutter modes. The limited validity of the energy compatibility equation in tracing the important flutter modes may be explained as follows. For the undamped system, the energy compatibility equation consists of the aerodynamic coefficients and the in-phase response (at flutter) only. Furthermore, the flutter speed for the undamped system shows a stationary property with respect to variations in the in-phase responses. This property implies that the flutter speed is insensitive to changes in modal responses. For the damped system, the stationary property does not hold and the flutter speed is therefore sensitive to changes in modal responses. Furthermore, the out-of-phase responses which now appear in the energy compatibility equation considerably complicate its form. The insensitivity of the flutter speed to changes in modal responses is important since the reduction of the fluttering system to a smaller order system is bound to be accompanied by changes in modal responses which, in turn, affect the energy compatibility equation. It can therefore be concluded that the energy compatibility method is valid for the detection of important flutter modes of undamped systems only.

A second energy method for the determination of the important flutter modes is suggested in Ref. 3. In this case, the energy input per cycle P is given by the following expression:

$$P = \tfrac{1}{2}\rho V^2 \sum_{i=1}^{n} \sum_{j=1}^{m} q_i^* U_{ij} q_j \tag{1}$$

where ρ denotes the fluid density, V the flight velocity, q_i the ith complex modal response at flutter, and $[U]$ a complex square Hermitian matrix of aerodynamic coefficients only ($[U]$ is a function of the Mach number M and of the reduced frequency of oscillation). The symbol * denotes the conjugate vector. Let:

$$E_{ij} = \tfrac{1}{2}\rho V^2 q_i^* U_{ij} q_j \tag{2}$$

then

$$P = \sum_{i=1}^{n} \sum_{j=1}^{m} E_{ij} \tag{3}$$

The energy method suggests that the dominant flutter modes are those associated with the values of i and j which have dominant E_{ij} terms [in Eq. (3)], implying large values of energy exchanges. Table V of Ref. 4, however, does not support the above suggestion. Many other examples have subsequently been computed which support the conclusion regarding the inability of the energy method to detect the important flutter modes. Here again, the reason for this inability lies in the changes which occur in the modal

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responses and modal frequencies of the system as a result of its reduction in order and which affect, in turn, the energy balance of the system at the flutter speed.

In the present paper, a new method is developed which enables the determination of the important flutter modes by avoiding the need to assume constant values for the modal responses and by avoiding the systematic recomputation of the flutter speeds of a large array of reduced-order systems.

Description of the Method

The n equations

$$([B]\sigma^2 + Q_D[A] + [E])q = 0 \tag{4}$$

represent the equations of motion of the system where $[B]$ represents the mass matrix, $[E]$ the stiffness matrix, $[A]$ the complex aerodynamic matrix, Q_D the flight dynamic pressure, and q the complex response of the system. The variable σ represents the eigenvalues of the system, that is

$$\sigma = \delta + i\omega \tag{5}$$

where ω is the frequency of oscillation and δ indicates the stability of oscillation (stable if values of $\delta < 0$) and $i = \sqrt{-1}$.

It is assumed that Eq. (4) is valid for a specific value of Mach number M. This enables the determination of the aerodynamic matrix $[A]$ and the value of the flight speed V (since the speed of sound varies only slightly with altitude). As mentioned previously, the complex, aerodynamic matrix is a function of the reduced frequency k. This dependence is made explicit by the following form of Pade approximants:

$$[A] = [A_0] + [A_1]ik + [A_2](ik)^2 + \sum_{j=1}^{r} \frac{[C_j](ik)}{(ik) + d_j} \tag{6}$$

where normally $r \leq 4$, d_j represents real constants, and the $[C_j]$, $[A_0]$, $[A_1]$, $[A_2]$ matrices are all real. With the value of V known, Eq. (6) can be brought to the form:

$$[A] = [\bar{A}_0] + [\bar{A}_1]\sigma + [\bar{A}_2]\sigma^2 + \sum_{j=1}^{r} \frac{[C_j]\sigma}{\sigma + d_j V/b} \tag{7}$$

For flutter to occur, the determinant D_n of the coefficients of order n of Eq. (4) must vanish while $\sigma = i\omega$. The vanishing of the subdeterminant D_{n-1} of Eq. (4) (obtained by ignoring a different single degree of freedom at a time) may form a convenient criterion for the detection of the important flutter modes since they contain no elements of the modal responses at flutter. However, the values of these subdeterminants show a considerable sensitivity to small variations in the values of Q_D and σ, and thus turn the method into an impractical one. To overcome this difficulty, one should allow limited variations in the values of the above two parameters so as to reduce D_{n-1} to zero or, alternatively, to minimize $D_{n-i}D^*_{n-1}$ by constraining σ to imaginary values only and by letting both Q_D and σ vary within a prescribed range of values. Once a minimum is obtained, it is necessary to note whether the variations in the values of ω and Q_D are indeed negligible compared to the values associated with flutter of the expanded system. In addition, it is necessary to check whether the minimum obtained is associated with flutter. This latter point is checked by reminimization of D_{n-1} with the new values of ω and Q_D kept constant and by letting δ assume values other than zero. Clearly, if the value of (δ/ω) thus obtained is small (less than say 0.01), then it may be considered as a numerical inaccuracy and the minimum obtained is interpreted as flutter. If, on the other hand, the value obtained for (δ/ω) is large, the minimum obtained indicates a stationary point which is not associated with flutter (it can be shown that $(\delta/\omega) = 0.5\ g$, where g represents structural damping coefficient). The ideas described above form the basis of the method proposed in this work. The details relating to the various stages are outlined by the following procedure:

1) For a chosen Mach number, the flutter frequency and the flutter dynamic pressure of the original (expanded) system are substituted in the equations of motion.

2) The quadratic value of the complex subdeterminant $D_{n-i}D^*_{n-1}$ (obtained by ignoring in the equations of motion a different single degree of freedom each time) is minimized by means of an optimization program which permits the variation of both the further dynamic pressure Q_D and the flutter frequency ω (with $\delta = 0$) within prescribed bounds.

3) The minimum value of the subdeterminant found in step 2 is further reduced by permitting δ to vary (keeping both Q_D and ω at the values obtained in step 2). The value of (δ/ω) thus obtained serves to indicate whether the minimum obtained in step 2 implies a limit of stability or, alternatively, a minimum within a damped region.

4) The modes, which when ignored lead to large changes in dynamic pressure or frequency, are selected as the important flutter modes, provided step 3 yields negligibly small values for (δ/ω).

5) The reduced system defined in step 4 is retested for possible further reduction by following a procedure identical to the one just described.

Presentation and Discussion of Results

Seven examples will be presented in order to indicate the effectiveness of the method just described. Five of the examples are identical to those appearing in Ref. 2 with a single exception—all the present examples relate to damped systems, whereas in Ref. 2 they all relate to undamped systems (by ignoring the overall damping matrix). The two additional examples relate to a supersonic cruise aircraft (SST) and to a fighter aircraft (YF-17).

The results for each of the above examples are presented in four different tables: The first table presents the natural frequencies of the different modes together with the eigenvalues $\delta + i\omega$ around the flutter dynamic pressure Q_{DF}. The second table shows the variations in the flutter velocity ΔV_F and the flutter frequency $\Delta\omega_F$ (both expressed in percentages) as a result of ignoring, in turn, one of the degrees of freedom of the system (following step 2 above). This table also shows the values of g (expressed in percentages) following step 3. The third table shows the results of step 5 where attempts are

Table 1 Flutter example of a delta wing with fixed root: Normalized natural frequencies and the normalized frequencies at flutter dynamic pressure Q_{DF}

Mode No. and type	Normalized natural frequency, rad/s ω_n	Normalized eigenvalues at flutter, rad/s σ_F	ω_F
Five modes of	0.49	0.00	0.63
fundamental	0.65	−0.04	0.54
and harmonic	1.88	−0.08	1.90
bending and	16.73	−0.07	16.72
torsion	5.22	−0.05	5.19

Table 2 Flutter example of a delta wing with fixed root: Effect on flutter velocity and flutter frequency of ignoring a different single mode each time, using the optimization procedure

	Mode No. ignored 1	2	3	4	5
ΔV_F, %	NF[a]	NF	0.0	0.0	0.0
$\Delta\omega_F$, %	NF	NF	0.0	0.0	0.0
g,% damping	NF	NF	0.0	0.0	0.0

[a] NF—No flutter within prescribed bounds of parametric variation using the optimization procedure.

Table 3 Flutter example of a delta wing with fixed root: Performance of the reduced system using the optimization procedure

Modes present in flutter analysis 1	2	ΔV_F, %	$\Delta\omega_F$, %	g, %
*	*	0.0	0.0	0.0

Table 4 Flutter example of a delta wing with fixed root: Normalized flutter velocities and normalized flutter frequencies of different reduced-order systems, using routine flutter program

Modes present in flutter analysis 1	2	ΔV_F, %	$\Delta\omega_F$, %
*	*	0.0	0.0

Table 5 Flutter example of the YF-17 fighter: Natural frequencies and the frequencies at flutter dynamic pressure Q_{DF}

Mode No.	Mode type	Natural frequency, rad/s ω_n	Eigenvalues at flutter, rad/s σ_F	ω_F
1	Rigid body plunge	0.0	—	—
2	Rigid body pitch	0.0	—	—
3	First bending	32.3	−0.94	34.3
4	First torsion	47.7	0.02	44.3
5	Second bending	91.2	−0.78	91.1
6	Third bending	102.3	−0.81	102.6
7	Fourth bending	231.2	−1.04	230.1
8	Second torsion	242.2	−0.08	241.7
9	Bending torsion	270.8	−0.45	266.1

Table 6 Flutter example of the YF-17 fighter: Effect on flutter velocity and flutter frequency of ignoring a different single mode each time, using the optimization procedure

	Mode ignored 1	2	3	4	5
ΔV_F, %	15.1	3.8	NF[a]	NF	0.2
$\Delta\omega_F$, %	−1.4	−0.3	NF	NF	0.0
g, % damping	0.0	0.0	NF	NF	0.0

	Mode ignored 6	7	8	9
ΔV_F, %	−5.7	−0.2	0.0	0.4
$\Delta\omega_F$, %	0.4	0.0	0.0	0.1
g, % damping	0.0	0.0	0.0	0.0

[a] NF—No flutter within prescribed bounds of parametric variation using the optimization procedure.

Table 7 Flutter example of the YF-17 fighter: Performance of the reduced system, using the optimization procedure

Modes present in flutter analysis 1	3	4	ΔV_F, %	$\Delta\omega_F$, %	g, %
*	*	*	−2.7	0.6	0.00
	*	*	3.4	−0.7	0.00

Table 8 Flutter example of the YF-17 fighter: Flutter velocities and flutter frequencies of different reduced-order systems, using routine flutter program

Modes present in flutter analysis 1	3	4	ΔV_F, %	$\Delta\omega_F$, %
*	*	*	−2.4	0.6
	*	*	4.1	−0.7

Table 9 Flutter example of the Sea-Venom aircraft: Normalized natural frequencies and the normalized frequencies at flutter dynamic pressure Q_{DF}

Mode No.	Mode type	Normalized natural frequency, rad/s ω_n	Normalized eigenvalues at flutter, rad/s σ_F	ω_F
1	Boom bending	1.12	−0.04	1.17
2	Tail mode	2.83	−0.01	2.83
3	Tailplane bending	3.43	0.01	3.26
4	Elevator rotation	0.00	−0.03	0.53
5	Spring tab angle	0.89	−0.16	2.05
6	Trim tab angle	2.19	−0.42	3.15

Table 10 Flutter example of the Sea-Venom aircraft: Effect on flutter velocity and flutter frequency of ignoring a different single mode each time, using the optimization procedure

	Mode ignored 1	2	3	4	5	6
ΔV_F, %	0.2	0.0	NF[a]	4.7	−0.5	NF
$\Delta\omega_F$, %	−0.3	0.1	NF	−2.6	0.5	NF
g, % damping	0.0	0.0	NF	0.0	0.0	NF

[a] NF—No flutter within prescribed bounds of parametric variation using the optimization procedure.

Table 11 Flutter example of the Sea-Venom aircraft: Performance of the reduced system using the optimization procedure

Modes present in flutter analysis 3	6	ΔV_F, %	$\Delta\omega_F$, %	g, %
*	*	−2.0	−2.2	0.0

Table 12 Flutter example of the Sea-Venon aircraft: Normalized flutter velocities and normalized flutter frequencies of different reduced-order systems, using routine flutter program

Modes present in flutter analysis 3	6	ΔV_F, %	$\Delta\omega_F$, %
*	*	2.0	−0.4

Table 13 Flutter example of a Handley Page 80/F/60 aircraft: Normalized natural frequencies and the normalized frequencies at flutter dynamic pressure Q_{DF}

Mode No.	Mode type	Normalized natural frequency, rad/s ω_n	Normalized eigenvalues at flutter, rad/s σ_F	ω_F
1	Antisymmetric wing fundamental bending	0.50	0.01	0.78
2	Antisymmetric wing harmonic bending	1.14	−0.20	0.77
3	Roll	0.00	−0.07	0.12
4	Aileron rotation	2.55	−0.07	2.73
5	Antisymmetric wing fundamental torsion	1.37	−0.03	1.26

Table 14 Flutter example of a Handley Page 80/F/60 aircraft: Effect on flutter velocity and flutter frequency of ignoring a different single mode each time, using the optimization procedure

	Mode ignored 1	2	3	4	5
ΔV_F, %	15.5	−1.4	−4.0	−3.0	15.5
$\Delta\omega_F$, %	−18.5	23.5	−9.8	4.4	−34.8
g, % damping	−7.3	−0.7	0.0	0.2	−13.2

Table 15 Flutter example of a Handley Page 80/F/60 aircraft: Performance of the reduced system and attempts to further reduce its order, using the optimization procedure

Modes present in flutter analysis 1	2	5	ΔV_F,%	$\Delta\omega_F$,%	g,%
*	*	*	−7.1	−5.4	0.1
	*	*	15.5	44.1	−6.5
*		*	−1.5	3.5	0.0
*	*		15.5	−32.9	−11.0

Table 16 Flutter example of a Handley Page 80/F/60 aircraft: Normalized flutter velocities and normalized flutter frequencies of different reduced-order systems, using routine flutter program

Modes present in flutter analysis 1	2	5	ΔV_F,%	$\Delta\omega_F$,%
*	*	*	−8.2	−12.4
	*	*	>30.0	—
*		*	−2.0	2.5
*	*		>30.0	—

Table 17 Flutter example of a hypothetical aircraft: Normalized natural frequencies and the normalized frequencies at flutter dynamic pressure Q_{DF}

Mode No.	Mode type	Normalized natural frequency, rad/s ω_n	Normalized eigenvalues at flutter, rad/s σ_F	ω_F
1	Parabolic bending of fuselage	3.30	0.00	3.22
2	Linear torsion of fuselage	4.66	−0.07	4.55
3	Starboard tailplane rotation about quarter chord	13.61	−0.04	13.53
4	Port tailplane rotation about quarter chord	11.14	−0.21	11.14
5	Elevator rotation	0.00	−0.78	1.98

Table 18 Flutter example of a hypothetical aircraft: Effect on flutter velocity and flutter frequency of ignoring a different single mode each time, using the optimization procedure

	Mode ignored 1	2	3	4	5
ΔV_F,%	NF[a]	8.0	0.4	0.2	NF
$\Delta\omega_F$,%	NF	−7.2	−0.40	0.2	NF
g,% damping	NF	0.0	0.0	0.0	NF

[a]NF—No flutter within prescribed bounds of parametric variation using the optimization procedure.

Table 19 Flutter example of a hypothetical aircraft: Performance of the reduced system and attempts to further reduce its order, using the optimization procedure

Modes present in flutter analysis 1	2	5	ΔV_F,%	$\Delta\omega_F$,%	g,%
*	*	*	0.5	−0.5	0.0
*	*		>30.0	—	—
*		*	8.5	−7.8	0.0
	*	*	−20.3	6.8	0.3

Table 20 Flutter example of a hypothetical aircraft: Normalized flutter velocities and normalized flutter frequencies of different reduced-order systems, using routine flutter program

Modes present in flutter analysis 1	2	5	ΔV_F,%	$\Delta\omega_F$,%
*	*	*	0.5	0.0
*		*	14.0	0.5

Table 21 Flutter example of the Handley Page Victor aircraft: Normalized natural frequencies and the normalized frequencies at flutter dynamic pressure Q_{DF}

Mode No.	Mode type	Normalized natural frequency, rad/s ω_n	Normalized eigenvalues at flutter, rad/s σ_n	ω_F
1	Fin bending	1.04	0.0	1.43
2	Wing bending	2.07	−0.05	2.04
3	Fuselage bending	2.09	−0.29	2.42
4	Elevator bending	3.60	−0.45	3.85
5	Elevator rotation	6.14	−0.56	5.54
6	Fin rotation	1.58	−0.49	1.73

Table 22 Flutter example of the Handley Page Victor aircraft: Effect on flutter velocity and flutter frequency of ignoring a different single mode each time, using the optimization procedure

	Mode ignored 1	2	3	4	5	6
ΔV_F,%	NF[a]	3.5	−2.7	NF	0.0	44.8
$\Delta\omega_F$,%	NF	−2.4	2.3	NF	0.8	−2.9
g,% damping	NF	0.0	0.0	NF	0.0	0.0

[a]NF—No flutter within prescribed bounds of parametric variation using the optimization procedure.

Table 23 Flutter example of the Handley Page Victor aircraft: Performance of the reduced system, and attempts to further reduce its order, using the optimization procedure

Modes present in flutter analysis 1	4	6	ΔV_F,%	$\Delta\omega_F$,%	g,%
*	*	*	0.8	0.4	0.0
	*	*	>30.0	—	—
*		*	>30.0	—	—
*	*		>30.0	—	—

Table 24 Flutter example of the Handley Page Victor aircraft: Normalized flutter velocities and normalized flutter frequencies of different reduced-order systems, using routine flutter program

Modes present in flutter analysis 1	4	6	ΔV_F,%	$\Delta\omega_F$,%
*	*	*	0.9	1.1

Table 25 Flutter example of the supersonic cruise aircraft (SST): Natural frequencies and the frequencies at flutter dynamic pressure Q_{DF}

Mode No.	Mode type	Natural frequency, rad/s ω_n	Eigenvalues at flutter, rad/s σ_F	ω_F
1	Rigid body plunge	0.0	0.0	0.0
2	Rigid body pitch	0.0	0.0	0.0
3	First bending	28.7	−0.77	27.6
4	Coupled bending torsion	51.8	−3.23	55.5
5	Coupled bending torsion	76.3	−4.66	78.7
6	Coupled bending torsion nacelles	94.3	0.05	89.6
7	Coupled bending torsion	112.9	−1.17	112.6
8	Second bending	121.6	−0.57	121.3
9	Coupled bending torsion	144.1	−1.10	152.8
10	Coupled bending torsion	150.2	−0.63	150.5
11	Coupled bending torsion	158.0	−0.18	157.9

Table 26 Flutter example of the supersonic cruise aircraft (SST):Effect on flutter velocity and flutter frequency of ignoring a different single mode each time, using the optimization procedure

	Mode ignored 1	2	3	4	5	6	7	8	9	10	11
ΔV_F,%	1.9	2.0	−0.4	22.7	34.0	NF[a]	−1.5	−0.7	5.0	0.0	0.0
$\Delta\omega_F$,%	−0.3	−0.2	−0.1	−2.3	−3.5	NF	0.1	0.1	−0.3	0.0	0.0
g,% damping	0.0	0.0	0.0	0.0	0.0	NF	0.0	0.0	0.0	0.0	0.0

[a] NF—No flutter within prescribed bounds of parametric variation using the optimization procedure.

Table 27 Flutter example of the supersonic cruise aircraft (SST): Performance of the reduced system and attempts to further reduce its order, using the optimization procedure

Modes present in flutter analysis 4	5	6	9	ΔV_F,%	$\Delta\omega_F$,%	g,%
*	*	*	*	1.9	−0.4	0.0
*	*	*		7.4	−0.7	0.0
	*	*		37.0	−3.4	0.0
*		*		>40.0	—	—
*	*			>40.0	—	—

Table 28 Flutter example of the supersonic cruise aircraft (SST): Flutter velocities and flutter frequencies of different reduced-order systems, using routine flutter program

Modes present in flutter analysis 4	5	6	9	ΔV_F,%	$\Delta\omega_F$,%
*	*	*	*	−0.8	−0.2
*	*	*		6.9	−0.6
*	*			>40.0	—
*		*		>40.0	—
	*	*		>40.0	—

made to reduce the system further. The fourth table shows the results obtained by employing a routine flutter program. These latter results are used for verification of the results obtained earlier by the optimization procedure developed in the present work.

The first flutter example relates to a delta wing with fixed root. The results obtained by the present method and by a routine flutter program are presented in Tables 1-4. It can be seen that the quintic flutter problem can be reduced to a binary flutter problem involving modes 1 and 2. This result could have been obtained, in this case, by the frequency coalescence method, as indicated by Table 1.

Another "obvious" example is presented in Tables 5-8 which relate to the YF-17 aircraft. The nine-degree-of-freedom system can be reduced to a binary flutter system involving modes 3 and 4. These modes are nearest to the flutter frequency and they could have been detected just by applying simple common sense (see Table 5).

The results of a third "obvious" example, which relates to the Sea-Venom aircraft, are presented in Tables 9-12. The six-degree-of-freedom system can be reduced to a binary flutter problem involving modes 3 and 6. Here again, these latter modes could have been easily detected using the frequency coalescence method.

The results of the fourth flutter example, which relates to the Handley Page 80/F/60 (Tables 13-16) are very interesting for two reasons. First, the quintic system can be reduced to a binary flutter problem involving modes 1 and 5. Table 13 shows frequency coalescence between modes 1 and 2 with a frequency value near the flutter frequency. The frequency of mode 5 is relatively far from the flutter frequency, yet it is a dominant flutter mode (together with mode 1). This example is indicative of the limitations of the frequency coalescence method, as mentioned earlier.

The results of the fifth flutter example, which relates to a hypothetical aircraft, are presented in Tables 17-20. Here, the quintic system is reduced to a ternary flutter problem involving modes 1,2, and 5. Table 19 shows that mode 2 is not absolutely essential. The importance of mode 5 cannot be detected from Table 17 (using the frequency coalescence method) and one might have guessed that modes 1 and 2 might be the dominant ones. This is clearly incorrect, as seen from Table 19.

The results of the sixth flutter example, which relates to the Handley Page Victor aircraft, are presented in Tables 21-24. Here, the six-degree-of-freedom system can be reduced to a ternary fluttering system involving modes 1, 4, and 6. It is interesting to note that none of the binary combinations yields a flutter instability. The importance of mode 4, with a frequency considerably higher than the flutter frequency, is difficult to predict using other methods.

The results for the last flutter example, which relates to an SST aircraft, are presented in Tables 25-28. It is interesting to note that modes 4, 5, 6, and 9 are responsible for flutter. If a

7.4% change in flutter speed is not considered to be large, the system can be further reduced to include modes 4, 5, and 6 only. This example is interesting since it shows, once again, that the basic flutter mechanism includes more than the usual two modes and that not even near coalescence of frequencies of the above dominant modes can be observed in Table 25.

It is interesting to note that in all the examples tested, the new optimization method for the detection of the important flutter modes yields the correct dominant modes.

Conclusion

The method developed for the determination of the important flutter modes shows excellent ability in the detection of the dominant flutter modes. The method is straightforward and is computationally efficient. It can therefore serve as a routine standard tool to help the aeroelastician to analytically overcome the flutter-type instabilities which appear within the flight envelope.

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